14.8 Eq. (14.9.3) is an approximation for when \( L \gg \Delta L \). The general expression should be \( w = \sqrt[4]{\frac{\lambda^2 L R_{\text{mirror}}^2}{\pi^2 \Delta L}} \). For part (b), you may use the approximation.

14.10 Find \( f^\theta \), \( w_o \), and \( z_o \) for each beam.

1. A laser cavity is composed of two concave end mirrors of radius \( R_1 = 100 \text{ cm} \) and \( R_2 = 50 \text{ cm} \).
   (a) Over what range of mirror separation is the cavity stable?
   (b) If the separation distance \( L \) is chosen to be 125cm, how far will the beam waist be from the 50cm mirror?
   (c) What will \( w_o \) and \( z_o \) be if \( \lambda = 1064 \text{ nm} \)?

2. A Gaussian laser beam is seen to have a 1cm intensity diameter (measured from the 1/e^2 points on each side of the beam). It is also observed to focus to a small spot 1m later. The wavelength is \( \lambda = 1064 \text{ nm} \). What are \( w_o \) and \( z_o \)?

3. A laser cavity is composed of two concave end mirrors of radius \( R_1 = 100 \text{ cm} \) and \( R_2 = \infty \). They are separated by a distance of \( L=50 \text{ cm} \). How far away from the flat mirror is the beam waist located? What are \( w_o \) and \( z_o \) if \( \lambda = 1064 \text{ nm} \)?
4. Show that \( E(\rho, z) = E_0 \frac{w_0}{w(z)} e^{-\frac{\rho^2}{w(z)^2}} e^{\frac{-i \tan^{-1} \frac{z}{z_0} + kp}{z_0^2 R(z)}} \) satisfies the paraxial wave equation

\[
\left( \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + 2ik \frac{\partial}{\partial z} \right) \tilde{E}(\rho, z) = 0.
\]

HINT: Be ready for a mess. It is not so bad if you write \( R(z) \) in terms of \( w(z) \) and use

\[
\frac{\partial w(z)}{\partial z} = \frac{w_0}{\sqrt{1 + z^2/z_0^2}} = \frac{w_0^2 z}{z_0 w(z)} = \frac{2z}{kz_0 w(z)} \quad \text{and} \quad \frac{\partial \tan^{-1} \frac{z}{z_0}}{\partial z} = \frac{1}{z_0(1 + z^2/z_0^2)} = \frac{w_0^2}{z_0 w^2(z)} = \frac{2}{kw^2(z)}.
\]

5. (a) How do the two terms in the paraxial approximation

\[
\left| 2k \frac{\partial \tilde{E}(\vec{r})}{\partial z} \right| \gg \left| \frac{\partial^2 \tilde{E}(\vec{r})}{\partial z^2} \right|
\]

data numerically compare at \( \rho = 0 \) and \( z = 0 \) when \( \lambda = 632\text{nm} \) and \( w_0 = 25\mu\text{m} \)? Assume a TEM\(_{00} \) mode.

(b) Suppose that we are satisfied if the above inequality at \( \rho = 0 \) and \( z = 0 \) is at least as strong as \( 100\gg1 \). What is the smallest \( f^\# \) that we can afford for a TEM\(_{oo} \) Gaussian mode? Note: In part (b) you are no longer assuming \( w_0 = 25\mu\text{m} \).

6. A laser beam has a waist of \( w_0 = 0.5\text{mm} \) as it enters a lens of \( f = 200\text{cm} \). Find the new beam waist \( w'_0 \) and its position after the lens if the wavelength is \( \lambda = 1064\text{nm} \).