Vector Addition Tutorial - Physics 105
Questions

Instructions: Answer each question on separate paper *before* looking at the answer frame. Then check your answer, make appropriate corrections, and follow the 'go to' instructions.

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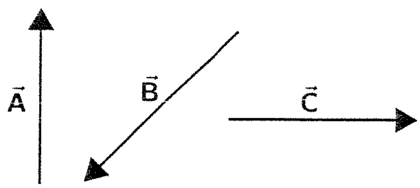
1. Here are three vectors: $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$.



In frames 2–12, sketch the required vector. Rough diagrams are sufficient.

Go to Frame 2

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2. Sketch $\frac{3}{2}\vec{\mathbf{C}} - \vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{D}}$.

3. Sketch $\vec{\mathbf{A}} + \vec{\mathbf{B}}$.

4. Sketch $\vec{\mathbf{C}} + \vec{\mathbf{B}}$.

5. Sketch $-\vec{\mathbf{B}}$.

6. Sketch $\vec{\mathbf{A}} - \vec{\mathbf{B}}$.

7. Sketch $\vec{\mathbf{B}} - \vec{\mathbf{C}}$.

8. Sketch $\vec{\mathbf{C}} - \vec{\mathbf{B}}$.

9. Sketch $3\vec{\mathbf{C}}$.

10. Sketch $\frac{1}{2}\vec{\mathbf{B}}$.

11. Sketch $-2\vec{\mathbf{C}}$.

12. Sketch $2\vec{\mathbf{A}} + \frac{3}{4}\vec{\mathbf{B}} - \frac{1}{3}\vec{\mathbf{C}} = \vec{\mathbf{R}}$.

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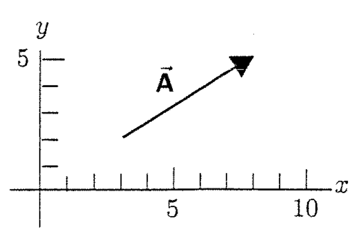
13. Here is a vector and a frame of reference. What are the rectangular components of the vector in this frame?



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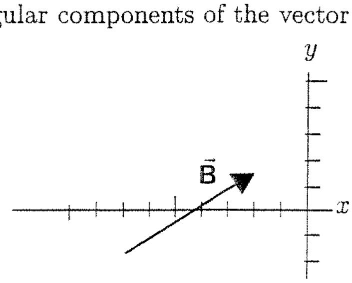
14. Here is a vector and a frame of reference. What are the rectangular components of the vector in this frame?



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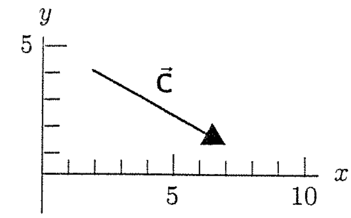
15. Here is a vector and a frame of reference. What are the rectangular components of the vector in this frame?



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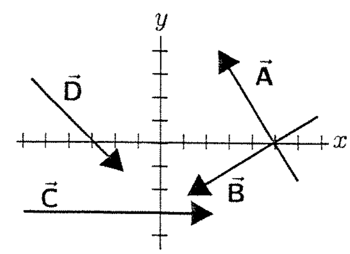
16. What are the rectangular components of these 4 vectors?



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17. Vector $\vec{\mathbf{A}}$ has magnitude 7 meters, is parallel to the xy plane, and makes is directed 230° counterclockwise from the positive x axis. What are the rectangular components of $\vec{\mathbf{A}}$?

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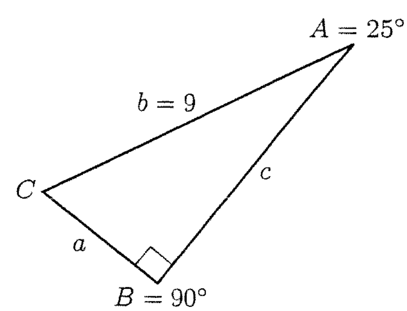
18. What are the values of the sides c and a in triangle ABC?



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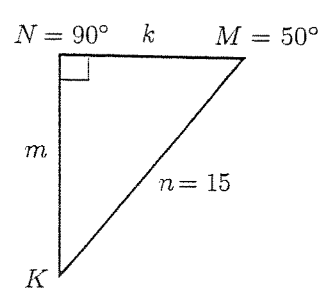
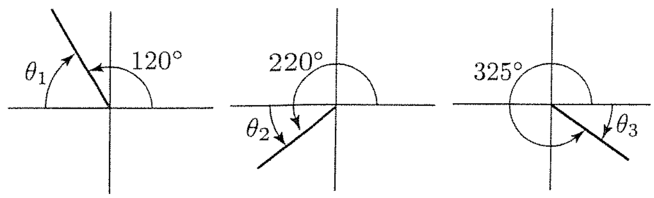
19. What are the values of sides k and m in triangle KMN?



20. What are the values of θ_1, θ_2 and θ_3 in the diagrams below?



21. An 8 N-force $\vec{F}$ is directed 160°counterclockwise from the direction of the positive x axis in the xy plane. Sketch an arrow representing $\vec{F}$, placing the tail of the vector at the origin. Also, show the rectangular components of $\vec{F}$ on your sketch.

22. Notice that, in the answer for frame 21, that triangle $m0n$ is a right triangle with its 3 sides proportional to the magnitudes of $\vec{F}$, F_x and F_y. Sketch this triangle, showing on the sketch the values of known quantities — one side and one acute angle.

23. Calculate the magnitudes of the rectangular components (F_x and F_y) of $\vec{F}$.

24. Next we assign algebraic signs to these components. Do so, and write the values of the rectangular components of $\vec{F}$.

25. A 15 m/s velocity $\vec{v}$ is directed 300°counterclockwise from the direction of the positive x axis in an xy plane. What are the rectangular components of this velocity?

26. A 3 N force is directed 225°counterclockwise from the direction of the positive x axis in an xy plane. What are the rectangular components of this force?

27. A force $\vec{F}$ has rectangular components $F_x = +3\,\text{N}$ and $F_y = -6\,\text{N}$. What are the polar components (the magnitude and direction) of $\vec{F}$?

28. A velocity $\vec{v}$ has rectangular components $v_x = -3\,\text{m/s}$ and $v_y = +5\,\text{m/s}$. Sketch a vector diagram showing the rectangular components of $\vec{v}$; then identify the polar components of $\vec{v}$.

29. Notice, in answer frame 28, that the triangle $m0n$ is a right triangle. Sketch the triangle, showing the values of known quantities (two sides) on the sketch.

30. Calculate the magnitude of $\vec{v}$ using the triangle in your answer to frame 29.

31. Calculate the value of the angle α in the triangle in answer frame 29.

32. Calculate the angle θ between the direction of $\vec{v}$ and the direction of the positive x axis.

33. Force $\vec{F}$ has rectangular components $F_x = +6\,\text{N}$ and $F_y = +4\,\text{N}$. Sketch a vector diagram showing the rectangular components of $\vec{F}$; then calculate the polar components (the magnitude and direction) of $\mathbf{F}$.

34. $\vec{a}$ has rectangular components $a_x = -5\,\text{m/s}^2$ and $a_y = -8\,\text{m/s}^2$. Sketch a vector diagram showing the rectangular components of $\vec{a}$; then calculate the polar components (the magnitude and direction) of $\vec{a}$.

35. Three forces are expressed in rectangular components as $\vec{F}_1 = (+3, -2)$ N, $\vec{F}_2 = (-5, +4)$ N, and $\vec{F}_3 = (+3, +4)$ N. Express the force $\vec{F}_4 = \vec{F}_1 - \vec{F}_2 + \vec{F}_3$ in rectangular form. $\vec{F} = (A, B)$ means $F_x = A$ and $F_y = B$. This is *matrix* notation.

36. Two displacements are expressed in rectangular components as $\vec{d}_1 = (3, -2)$ m and $\vec{d}_2 = (-5, +4)$ m. Express $\vec{d}_3 = \vec{d}_1 + \vec{d}_2$ in rectangular form.

37. Using $\vec{d}_1$ and $\vec{d}_2$ from frame 36, express $\vec{d}_4 = \vec{d}_2 - \vec{d}_1$ in rectangular form.

38. $\vec{a} = (3, -4)$ m/s^2. Identify the x and y components of $\vec{a}$.

39. $F_x = -3\,\text{N}$ and $F_y = +4\,\text{N}$. Write $\vec{F}$ in matrix notation.

40. What is the magnitude F of the force in frame 39?

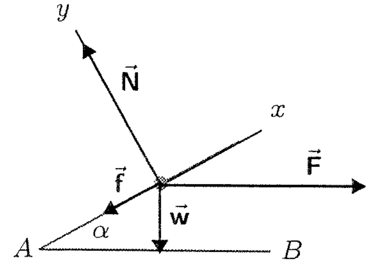
41. $\vec{a} = (3, -5)$ m. Express $3\vec{a}$ in unit vector in component form and in matrix form.

42. $\vec{b} = (-2, -3)$ m. Express $\frac{1}{2}\vec{b}$ in component form and in matrix form.

43. Using $\vec{a}$ and $\vec{b}$ from frames 41 and 42, express $\vec{c} = \vec{a} + \vec{b}$ and $\vec{d} = \vec{a} - \vec{b}$ in unit vector and in matrix form.

44. $\vec{r} = (-2, +4)$ N, $\vec{s} = (+8, -2)$ N and $\vec{t} = (-4, +6)$ N. Express $\vec{v} = \vec{s} - \vec{r} + \vec{t}$ in component form and in matrix form.

45. Here are four vectors, $\vec{N}$, $\vec{F}$, $\vec{w}$, and $\vec{f}$. $\vec{F}$ is parallel to the line AB and $\vec{w}$ is perpendicular to it. Construct a sketch showing the vector addition of these vectors and their sum $\vec{S} = \vec{N} + \vec{F} + \vec{w} + \vec{f}$.



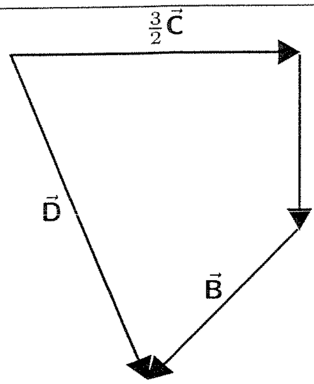
46. Now write expressions for the four vectors in frame 45 in terms of rectangular components with coordinate axes as shown in the figure. Your results should be in terms of the vector magnitudes N, F, w, f and trig functions of the angle α.

47. Add the vectors in frame 46 to get expressions for the components S_x and S_y of their vector sum.

Answers

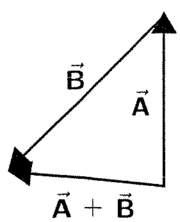
Instructions: Answer each question on separate paper *before* looking at these answers. Then check your answer, make appropriate corrections, and follow the 'go to' instructions.

2.



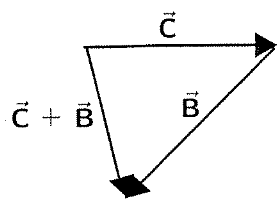
If ok, go to 12; if not, go to 3.

3.



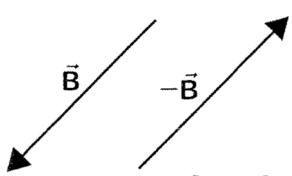
Go to 4.

4.



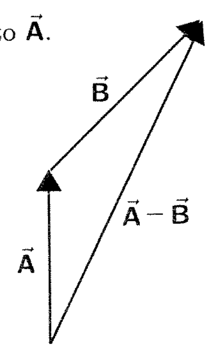
Go to 5.

5.



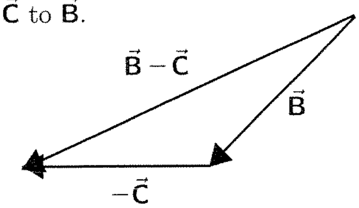
Go to 6.

6. Add $-\vec{B}$ to $\vec{A}$.



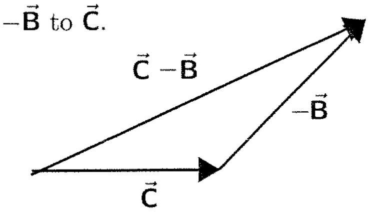
Go to 7.

7. Add $-\vec{C}$ to $\vec{B}$.



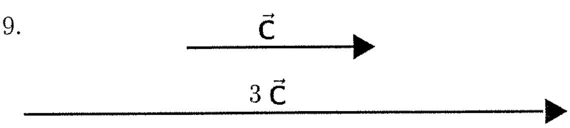
Go to 8.

8. Add $-\vec{B}$ to $\vec{C}$.



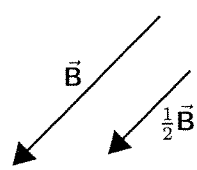
Go to 9.

9.



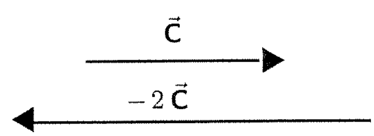
Go to 10.

10.



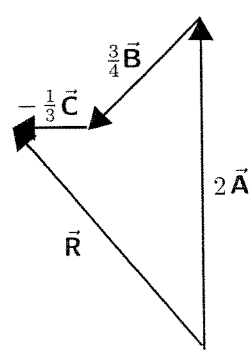
Go to 11.

11.



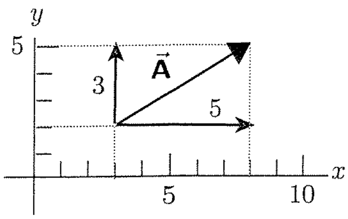
Go to 12.

12.



If ok go to 13; If not, go back to 3 or get help.

13. $A_x = +5$, $A_y = +3$



Notice how the magnitude of a component can be estimated by projecting the vector to the corresponding axis.

Go to 14.

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14. $A_x = +5$, $A_y = +3$

This is the same vector as in 13. Notice that the position of the vector does not affect its components.

Go to 15.

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15. $C_x = +5$, $C_y = -3$. The y component of a vector is negative if the vector points in the negative y direction.

Go to 16.

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16. $A_x = -3.5$, $A_y = +6$, $B_x = -6$, $B_y = -3$, $C_x = +8$, $C_y = 0$, $D_x = +4$, $D_y = -4$

The algebraic signs (+ or −) are an important part of these answers. Get help if you do not see how they are chosen.

If ok, go to 17; if not, get help.

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17. $A_x = -4.5$ m; $A_y = -5.36$ m.

If ok, go to 25; if not, go to 18.

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18. $c = 8.16$; $a = 3.80$

Apply the definitions of sine and cosine to the right triangle ABC: $\sin A = \dfrac{\text{opposite side}}{\text{hypoteneuse}} = \dfrac{a}{b}$.

Thus $a = b \sin A = 9 \sin 25° = 3.80$

Also, $\cos A = \dfrac{\text{adjacent side}}{\text{hypoteneuse}} = \dfrac{c}{b}$.

Thus $c = b \cos A = 9 \cos 25° = 8.16$

If ok, go to 20; if not, go to 19.
Get help if this is your second try.

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19. $m = 11.5$; $k = 9.64$.

$\sin M = \dfrac{m}{n}$; so $m = n \sin M = 15 \sin 50° = 11.5$

$\cos M = \dfrac{k}{n}$; so $k = n \sin M = 15 \cos 50° = 9.64$

If ok, go to 20; if not, get help.

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20. $\theta_1 = 60°$; $\theta_2 = 40°$; $\theta_3 = 35°$.

$\theta_1 = 180° - 120° = 60°$
$\theta_2 = 220° - 180° = 40°$
$\theta_3 = 360° - 325° = 35°$

If ok, go to 21; if not, get help.

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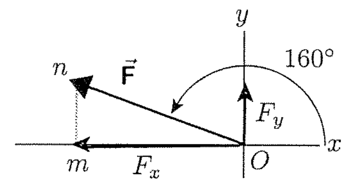
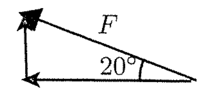
21. Your sketch should look something like this, except for the letters m and n which will be used in 21.



Go to 22.

22.



Go to 23.

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23. $|F_x| = 7.52\,\text{N}$; $|F_y| = 2.74\,\text{N}$.

$|F_x| = (8\,\text{N}) \cos 20° = 7.52\,\text{N}$;
$|F_y| = (8\,\text{N}) \sin 20° = 2.74\,\text{N}$

Go to 24.

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24. F_x is negative, so $F_x = -7.52\,\text{N}$
F_y is positive, so $F_y = +2.74\,\text{N}$

The problem of calculating rectangular components can always, in two dimensions, be reduced to the problem of calculating the sides of a right triangle. The algebraic signs are then assigned by analyzing the direction of the vector as in frames 15 and 16.

These calculations are done automatically by the 'polar to rectangular' (P→R) conversions on many calculators. If you use it, you should always do a quick sketch of the vector (like the one in answer frame 21) to check your calculated results.

If ok, go back to 17; if not, review frames 13-16.

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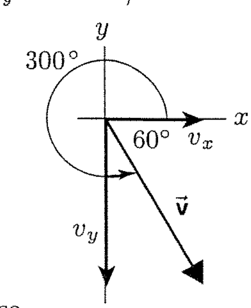
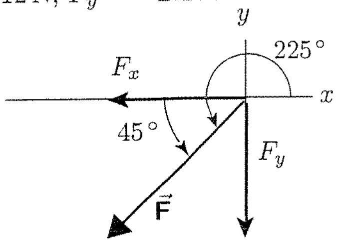
25. $v_x = +7.5\,\text{m/s}$; $v_y = -13\,\text{m/s}$



Velocity is a vector, so
$v_x = +v \cos 60° = (15\,\text{m/s}) \cos 60° = +7.5\,\text{m/s}$
$v_y = -v \sin 60° = -(15\,\text{m/s}) \sin 60° = -13\,\text{m/s}$

If ok, go to 27; if not go to 26.

26. $F_x = -2.12\,\mathrm{N}$; $F_y = -2.12\,\mathrm{N}$.



Force is a vector quantity, so
$F_x = -F\cos 45° = -2.12\,\mathrm{N}$; $F_y = -F\sin 45° = -2.12\,\mathrm{N}$

If ok, go to 27; if not, get help.

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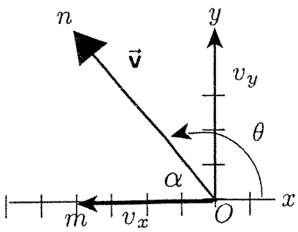
27. $F = 6.71$ N, $\theta_F = 296.5°$ (or $-63.5°$)

If ok, go to 35; if not, go to 28.

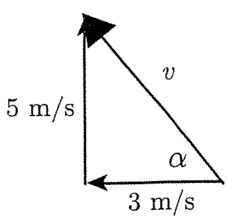
28.



v and θ are the polar components of $\vec{\mathbf{v}}$. The angle α is used in frames 29-33.

Go to 29.

29.



Go to 30.

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30. $A = \sqrt{34}\,\mathrm{m/s} = 5.83\,\mathrm{m/s}$

Using the Pythagorean Theorem,
$v^2 = v_x^2 + v_y^2 = (-3\,\mathrm{m/s})^2 + (5\,\mathrm{m/s})^2 = 34\,\mathrm{m^2/s^2}$.
So $v = \sqrt{34}\,\mathrm{m/s} = 5.83\,\mathrm{m/s}$

Go to 31.

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31. $\alpha = 59°$

$$\tan\alpha = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{5\,\mathrm{m/s}}{3\,\mathrm{m/s}} = 1.667.$$

$\alpha = \tan^{-1}(5/3) = 59°$.

Go to 32.

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32. $\theta = 180° - 59° = 121°$

You can always find the polar components by solving a right triangle.

You can also use the 'rectangular to polar' (R→P) conversion on many hand calculators. Again, you would be wise to construct a quick sketch, such as that in answer frame 28, to check the calculator results.

If ok, go to 33; if not, get help.

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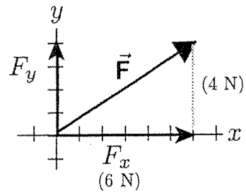
33. $\vec{\mathbf{F}} = (F, \theta) = (7.2\,\mathrm{N}, 33.7°)$



$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(4\,\mathrm{N})^2 + (6\,\mathrm{N})^2} = \sqrt{52}\,\mathrm{N} = 7.2\,\mathrm{N}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \left(\frac{4\,\mathrm{N}}{6\,\mathrm{N}}\right) = 33.7°.$$

Go to 34.

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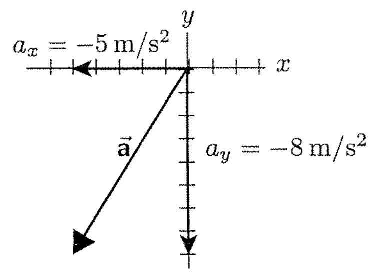
34. $9.4\,\mathrm{m/s^2}$, $238°$.



$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-5\,\mathrm{m/s^2})^2 + (-5\,\mathrm{m/s^2})^2}$$
$$= \sqrt{89}\,\mathrm{m/s^2} = 9.4\,\mathrm{m/s^2}.$$

$$\alpha = \tan^{-1}\left(\frac{|a_y|}{|a_x|}\right) = \tan^{-1}\left(\frac{8}{5}\right) = 58°.$$

$\theta = 180° + \alpha = 238°$. This shows the same strategy applied to a third quadrant vector. In this case, the hand calculator is much easier.

If ok, go to 35; if not, get help.

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35. $\vec{\mathbf{F}}_4 = (+11, -2)\ N$.

If ok, go to 38; if not, go to 36
(Get help if this is your second try)

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36. $\vec{\mathbf{d}}_3 = (-2, +2)\,\mathrm{m}$; $d_x = -2\,\mathrm{m}$, $d_y = +2\,\mathrm{m}$.

These additions are easiest to do if the vectors are vertically aligned as in a normal (scalar) addition. We show it here in matrix notation.

$$\begin{aligned} \vec{\mathbf{d}}_1 &= (+3, -2)\,\mathrm{m} \\ +\vec{\mathbf{d}}_2 &= (-5, +4)\,\mathrm{m} \\ \hline \vec{\mathbf{d}}_3 &= (-2, +2)\,\mathrm{m} \end{aligned}$$

Each component of the sum is just the sum of the corresponding components of the vectors being added.

Go to 37.

37. $\vec{\mathbf{d}}_4 = (-8, +6)\,\text{m}$; $d_x = -8\,\text{m}$, $d_y = +6\,\text{m}$.
Proceding as in frame 36,

$$\begin{aligned} \vec{\mathbf{d}}_2 &= (-5, +4)\,\text{m} \\ -\vec{\mathbf{d}}_1 &= \underline{(-3, +2)\,\text{m}} \\ \vec{\mathbf{d}}_4 &= (-8, +6)\,\text{m} \end{aligned}$$

The components of $-\vec{\mathbf{d}}_1$ are the negatives of the corresponding components of $\vec{\mathbf{d}}_1$.

Go to 35.

38. $a_x = +3\,\text{m/s}^2$; $a_y = -4\,\text{m/s}^2$.

Go to 39.

39. $\vec{\mathbf{F}} = (-3, +4)\,\text{N}$

Go to 40.

40. $F = 5\,\text{N}$

$$\begin{aligned} F &= \sqrt{F_x^2 + F_y^2} \\ &= \sqrt{(-3\,\text{N})^2 + (+4\,\text{N})^2} \\ &= \sqrt{25}\,\text{N} = 5\,\text{N} \end{aligned}$$

Notice that $(-3)^2 = +9$.

Go to 41.

41. $3\vec{\mathbf{a}} = (+9, -15)\,\text{m}$; $(3\vec{\mathbf{a}})_x = +9\,\text{m}$, $(3\vec{\mathbf{a}})_y = -15\,\text{m}$,

$$3\vec{\mathbf{a}} = 3(3, -5)\,\text{m} = (9, -15)\,\text{m}$$

Each component is multiplied by the scalar multiplier.

If ok, go to 43; if not, go to 42.

42. $\frac{1}{2}\vec{\mathbf{b}} = (-1, -1.5)\,\text{m}$. $(\frac{1}{2}\vec{\mathbf{a}})_x = -1\,\text{m}$, $(\frac{1}{2}\vec{\mathbf{a}})_y = -1.5\,\text{m}$,

If ok, go to 43; if not, get help.

43. $\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} = (+1, -8)\,\text{m}$; $c_x = 1\,\text{m}$, $c_y = -8\,\text{m}$.
$\vec{\mathbf{d}} = \vec{\mathbf{a}} - \vec{\mathbf{b}} = (+5, -2)\,\text{m}$; $d_x = +5\,\text{m}$, $d_y = -2\,\text{m}$.

$$\begin{aligned} \vec{\mathbf{a}} &= (+3, -5)\,\text{m} \\ \vec{\mathbf{b}} &= \underline{(-2, -3)\,\text{m}} \\ \vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} &= (+1, -8)\,\text{m} \end{aligned}$$

$$\begin{aligned} \vec{\mathbf{a}} &= (+3, -5)\,\text{m} \\ -\vec{\mathbf{b}} &= \underline{(+2, +3)\,\text{m}} \\ \vec{\mathbf{d}} = \vec{\mathbf{a}} - \vec{\mathbf{b}} &= (+5, -2)\,\text{m} \end{aligned}$$

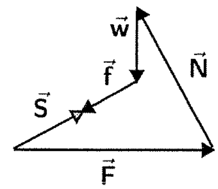
Go to 44.

44. $\vec{\mathbf{v}} = \vec{\mathbf{s}} - \vec{\mathbf{r}} + \vec{\mathbf{t}} = (+6, +0)\,\text{N}$; $v_x = +6\,\text{N}$, $v_y = 0\,\text{N}$.

$$\begin{aligned} \vec{\mathbf{s}} &= (+8, -2)\,\text{N} \\ -\vec{\mathbf{r}} &= (+2, -4)\,\text{N} \\ \vec{\mathbf{t}} &= \underline{(-4, +6)\,\text{N}} \\ \vec{\mathbf{v}} = \vec{\mathbf{s}} - \vec{\mathbf{r}} + \vec{\mathbf{t}} &= (+6, +0)\,\text{N} \end{aligned}$$

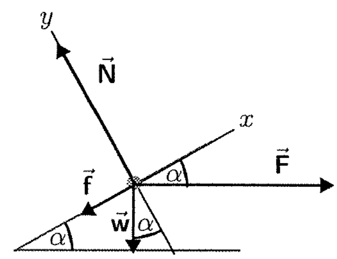
Go to 45.

45.



Go to 46.

46. Most people need to identify the angles between the vectors and the coordinate axes to do these kinds of analyses. Here is the same diagram as in fram 45 with the angles labeled.



The hardest one for most students is the angle between $\vec{\mathbf{w}}$ and the negative y axis. In is α, the same as the incline angle; be sure you see why this is true. With this, the vectors can be written in rectangular form as $\vec{\mathbf{N}} = (0, N)$, $\vec{\mathbf{f}} = (-f, 0)$, $\vec{\mathbf{F}} = (F\cos\alpha, -F\sin\alpha)$, and $\vec{\mathbf{w}} = (-w\sin\alpha, -w\cos\alpha)$.

Go to 47.

47. The components of the vector sum are
$\vec{\mathbf{S}}_x = -f + F\cos\alpha - w\sin\alpha$ and
$\vec{\mathbf{S}}_y = N - F\sin\alpha - w\cos\alpha$.

That's all folks!